

# D-branes from Pure Spinor Superstring in $\text{AdS}_5 \times \text{S}^5$ Background

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## Abstract

We examine the surface term for the BRST transformation of the open pure spinor superstring in an  $\text{AdS}_5 \times \text{S}^5$  background. We find that the boundary condition to eliminate the surface term leads to a classification of possible configurations of 1/2 supersymmetric D-branes.

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## 1 Introduction

Before the pure spinor formulation of the superstring was initiated by Berkovits [1], there were mainly two superstring formulations, a Ramond-Neveu-Schwarz (RNS) formulation and a Green-Schwarz (GS) formulation. The RNS superstring is described by a superconformal field theory on the two-dimensional world-sheet. So it is difficult to read off the target space geometry coupling to Ramond-Ramond fields, because spacetime supersymmetry emerges only after the GSO projection. On the other hand, for the GS superstring, spacetime supersymmetry is manifest from the outset. The action has world-sheet fermionic gauge symmetry, called  $\kappa$ -symmetry, instead of world-sheet supersymmetry. This makes it difficult to covariantly quantize the GS superstring even in flat spacetime. In the pure spinor superstring<sup>‡</sup>, the  $\kappa$ -symmetry in the GS superstring is replaced with the BRST symmetry. The pure spinor superstring can be quantized in a super-Poincaré covariant manner.

Furthermore, the pure spinor superstring in an  $\text{AdS}_5 \times \text{S}^5$  background with Ramond-Ramond flux [1] is shown to be consistent even at the quantum level [5, 6]. The action

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<sup>‡</sup> The extended versions of the pure spinor superstring were proposed in [2, 3] which introduced new ghosts to relax the pure spinor constraint, and in [4] which introduced doubled spinor degrees of freedom with a compensating local supersymmetry.

is composed of  $\mathfrak{psu}(2, 2|4)$  currents  $J$ , and the left- and right-moving ghosts,  $(\lambda^\alpha, w_\alpha)$ , and  $(\hat{\lambda}^{\hat{\alpha}}, \hat{w}_{\hat{\alpha}})$ , respectively. The ghosts satisfy the pure spinor constraints  $\lambda\gamma^A\lambda = \hat{\lambda}\gamma^A\hat{\lambda} = 0$  ( $A = 0, 1, \dots, 9$ ). The pure spinor superstring in the  $\text{AdS}_5 \times \text{S}^5$  background, as well as the GS superstring in the  $\text{AdS}_5 \times \text{S}^5$  background given in [7], is integrable in the sense that infinitely many conserved charges are constructed [8, 9] (see also [10]). Nevertheless, for the detailed study of the AdS/CFT correspondence [11], covariant quantization of the superstring should be useful. Though the action of the pure spinor superstring in the  $\text{AdS}_5 \times \text{S}^5$  background is bilinear in the current  $J$ , its quantization is still difficult because the  $J$  is not (anti-)holomorphic unlike the principal chiral model. We need more effort to quantize the pure spinor superstring covariantly.

The purpose of this paper is to study D-branes in the  $\text{AdS}_5 \times \text{S}^5$  background. A D-brane is a solitonic object in string theory, and is characterized by the Dirichlet boundary condition of an open string. The classical BRST invariance of the open pure spinor superstring in a background implies that the background fields satisfy full non-linear equations of motion for a supersymmetric Born-Infeld action [12]. This is the open string version of [13] in which the classical BRST invariance of the closed pure spinor superstring in a curved background was shown to imply that the background fields satisfy full non-linear equations of motion for the type-II supergravity. For D-branes in the  $\text{AdS}_5 \times \text{S}^5$  background, supersymmetric D-brane configurations are derived in [14] by examining equations of motion for a Dirac-Born-Infeld action for each D-brane embedding ansatz.

In the present paper, we will examine D-branes in the  $\text{AdS}_5 \times \text{S}^5$  background by using the open pure spinor superstring. Especially, we concentrate ourselves on the BRST invariance in the presence of the boundary. Namely, we examine the surface term for the BRST transformation of the open pure spinor superstring in the  $\text{AdS}_5 \times \text{S}^5$  background. We will find that the boundary condition to eliminate the surface term leads to a classification of possible configurations of 1/2 supersymmetric D-branes. This approach is the pure spinor superstring version of [15, 18]<sup>§</sup>. In [15, 18], the boundary condition for the  $\kappa$ -symmetry surface term of the GS superstring in the  $\text{AdS}_5 \times \text{S}^5$  background was shown to lead to a classification of possible configurations of 1/2 supersymmetric D-branes. We find that our result is consistent with those obtained in [14, 15, 18]. One of the main advantages in our approach is that the derivation is much simpler than the one by using the Dirac-Born-Infeld

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<sup>§</sup> A covariant approach to study D-branes in flat spacetime was proposed by Lambert and West in [19].

action and the GS superstring action. This is because the pure spinor superstring action is bilinear in the currents, and because we don't need to deal with the  $\kappa$ -symmetry variation which is highly non-linear.

This paper is organized as follows. In section 2, after introducing the pure spinor superstring in the  $\text{AdS}_5 \times \text{S}^5$  background, we examine the BRST invariance of the open superstring action and extract the surface term. For the BRST invariance to be preserved even in the presence of the boundary, the surface term must be eliminated by a certain boundary condition. In section 3, we fix the boundary conditions by examining a few terms contained in the surface term. The boundary conditions lead us to a classification of possible 1/2 supersymmetric D-branes in the  $\text{AdS}_5 \times \text{S}^5$  background. In section 4, the boundary conditions fixed above are shown to eliminate all terms contained in the surface term. The last section is devoted to a summary and discussions. Our notation and convention are summarized in Appendix.

## 2 Pure Spinor Superstring in $\text{AdS}_5 \times \text{S}^5$ background

The manifestly covariant action of the pure spinor superstring in the  $\text{AdS}_5 \times \text{S}^5$  background [1, 5, 16] (see e.g. [17] for reviews) is composed of three parts

$$S = S_\sigma + S_{\text{WZ}} + S_{\text{gh}} , \quad (2.1)$$

with

$$S_\sigma = \frac{1}{2} \langle J_2 \bar{J}_2 + J_1 \bar{J}_3 + J_3 \bar{J}_1 \rangle , \quad (2.2)$$

$$S_{\text{WZ}} = \frac{1}{4} \langle J_3 \bar{J}_1 - J_1 \bar{J}_3 \rangle , \quad (2.3)$$

$$S_{\text{gh}} = \left\langle w \bar{\partial} \lambda + \hat{w} \partial \hat{\lambda} + N \bar{J}_0 + \hat{N} J_0 - N \hat{N} \right\rangle , \quad (2.4)$$

where  $\langle \cdots \rangle$  stands for  $\frac{1}{\pi \alpha'} \int d^2 \sigma \text{Str}(\cdots)$ . Here  $J = \frac{1}{2}(J_\tau + J_\sigma)$  and  $\bar{J} = \frac{1}{2}(J_\tau - J_\sigma)$ , are the left- and right-moving currents, respectively. The  $J_\xi$  ( $\xi = \tau, \sigma$ ) stands for the pullback of the Cartan one-form on the worldsheet,  $J_\xi = g^{-1} \partial_\xi g$ , with  $g \in \text{PSU}(2, 2|4)/(\text{SO}(1, 4) \times \text{SO}(5))$ . Furthermore  $J_\xi$  is decomposed into four parts, under the  $\mathbb{Z}_4$ -graded decomposition of  $\mathfrak{psu}(2, 2|4)$ , namely

$$\begin{aligned} J_\xi &= J_{0\xi} + J_{1\xi} + J_{2\xi} + J_{3\xi}, \\ J_{0\xi} &= \frac{1}{2} J_{0\xi}^{ab} M_{ab} + \frac{1}{2} J_{0\xi}^{a'b'} M_{a'b'} , \quad J_{1\xi} = q_\alpha J_{1\xi}^\alpha , \quad J_{2\xi} = J_{2\xi}^a P_a + J_{2\xi}^{a'} P_{a'} , \quad J_{3\xi} = \hat{q}_{\hat{\alpha}} J_{3\xi}^{\hat{\alpha}} . \end{aligned} \quad (2.5)$$

The set of generators  $\{M_{ab}, M_{a'b'}, P_a, P_{a'}, q_\alpha, \hat{q}_{\hat{\alpha}}\}$  of  $\mathfrak{psu}(2, 2|4)$  satisfies (anti-)commutation relations given in (A.1) and (A.14). The pure spinor variables are defined as

$$\lambda = \lambda^\alpha q_\alpha, \quad \hat{\lambda} = \hat{\lambda}^{\hat{\alpha}} \hat{q}_{\hat{\alpha}}, \quad w = w_\alpha (\tilde{\gamma}^{0\dots 4})^{\alpha\hat{\alpha}} \hat{q}_{\hat{\alpha}}, \quad \hat{w} = \hat{w}_{\hat{\alpha}} (\tilde{\gamma}^{0\dots 4})^{\hat{\alpha}\alpha} q_\alpha, \quad (2.6)$$

where  $(\lambda^\alpha, w_\alpha)$  and  $(\hat{\lambda}^{\hat{\alpha}}, \hat{w}_{\hat{\alpha}})$  are left- and right-moving ghosts, respectively. In terms of these ghosts the Lorentz currents are given as  $N = -\{w, \lambda\}$  and  $\hat{N} = -\{\hat{w}, \hat{\lambda}\}$ .

## 2.1 BRST invariance

The BRST transformation of the action is examined below. We will not drop any surface term here. In the next section we will consider the boundary condition for the surface term to be eliminated, and show that the condition leads us to a classification of possible configurations of 1/2 supersymmetric D-branes in the  $\text{AdS}_5 \times \text{S}^5$  background.

First we examine  $S_\sigma$ . The BRST transformation law of currents with a Grassmann odd parameter  $\varepsilon$  is given as [5]

$$\begin{aligned} \varepsilon Q(J_1) &= \nabla(\varepsilon\lambda) + [J_2, \varepsilon\hat{\lambda}], & \varepsilon Q(\bar{J}_1) &= \bar{\nabla}(\varepsilon\lambda) + [\bar{J}_2, \varepsilon\hat{\lambda}], \\ \varepsilon Q(J_2) &= [J_1, \varepsilon\lambda] + [J_3, \varepsilon\hat{\lambda}], & \varepsilon Q(\bar{J}_2) &= [\bar{J}_1, \varepsilon\lambda] + [\bar{J}_3, \varepsilon\hat{\lambda}], \\ \varepsilon Q(J_3) &= \nabla(\varepsilon\hat{\lambda}) + [J_2, \varepsilon\lambda], & \varepsilon Q(\bar{J}_3) &= \bar{\nabla}(\varepsilon\hat{\lambda}) + [\bar{J}_2, \varepsilon\lambda], \\ \varepsilon Q(J_0) &= [J_3, \varepsilon\lambda] + [J_1, \varepsilon\hat{\lambda}], & \varepsilon Q(\bar{J}_0) &= [\bar{J}_3, \varepsilon\lambda] + [\bar{J}_1, \varepsilon\hat{\lambda}], \end{aligned} \quad (2.7)$$

where  $\nabla A \equiv \partial A + [J_0, A]$  and  $\bar{\nabla} A \equiv \bar{\partial} A + [\bar{J}_0, A]$ . By using (2.7), we obtain

$$\varepsilon Q(S_\sigma) = \frac{1}{2} \left\langle J_1 \bar{\nabla}(\varepsilon\hat{\lambda}) + \nabla(\varepsilon\lambda) \bar{J}_3 + J_3 \bar{\nabla}(\varepsilon\lambda) + \nabla(\varepsilon\hat{\lambda}) \bar{J}_1 \right\rangle. \quad (2.8)$$

To derive this expression we have used the cyclicity of the Str, for example  $\text{Str}(J_2[\bar{J}_1, \varepsilon\lambda]) = \text{Str}(\varepsilon\lambda[J_2, \bar{J}_1])$ .

Next we consider  $S_{\text{WZ}}$ . Using (2.7), one derives

$$\begin{aligned} \varepsilon Q(S_{\text{WZ}}) &= \frac{1}{4} \left\langle \nabla(\varepsilon\hat{\lambda}) \bar{J}_1 + J_3 \bar{\nabla}(\varepsilon\lambda) - \nabla(\varepsilon\lambda) \bar{J}_3 - J_1 \bar{\nabla}(\varepsilon\hat{\lambda}) \right. \\ &\quad \left. + \varepsilon\lambda([\bar{J}_1, J_2] - [J_1, \bar{J}_2]) + \varepsilon\hat{\lambda}([J_3, \bar{J}_2] - [\bar{J}_3, J_2]) \right\rangle. \end{aligned} \quad (2.9)$$

By using Maurer-Cartan equations

$$\nabla \bar{J}_3 - \bar{\nabla} J_3 = [\bar{J}_1, J_2] - [J_1, \bar{J}_2], \quad \nabla \bar{J}_1 - \bar{\nabla} J_1 = [\bar{J}_3, J_2] - [J_3, \bar{J}_2], \quad (2.10)$$

the second line of the right-hand side of (2.9) may be rewritten as

$$\begin{aligned} \varepsilon Q(S_{\text{WZ}}) = & \frac{1}{4} \left\langle \nabla(\varepsilon \hat{\lambda}) \bar{J}_1 + J_3 \bar{\nabla}(\varepsilon \lambda) - \nabla(\varepsilon \lambda) \bar{J}_3 - J_1 \bar{\nabla}(\varepsilon \hat{\lambda}) \right. \\ & \left. + \varepsilon \lambda (\nabla \bar{J}_3 - \bar{\nabla} J_3) - \varepsilon \hat{\lambda} (\nabla \bar{J}_1 - \bar{\nabla} J_1) \right\rangle . \end{aligned} \quad (2.11)$$

Finally we examine  $S_{\text{gh}}$ . The BRST transformation law of ghosts

$$\varepsilon Q(w) = -J_3 \varepsilon , \quad \varepsilon Q(\hat{w}) = -\bar{J}_1 \varepsilon , \quad \varepsilon Q(\lambda) = \varepsilon Q(\hat{\lambda}) = 0 \quad (2.12)$$

implies that

$$\varepsilon Q(N) = [J_3, \varepsilon \lambda] , \quad \varepsilon Q(\hat{N}) = [\bar{J}_1, \varepsilon \hat{\lambda}] . \quad (2.13)$$

Further noting that

$$\text{Str}(N[\bar{J}_3, \varepsilon \lambda]) = \text{Str}(-\bar{J}_3 \varepsilon [\lambda \lambda, w]) = 0 , \quad \text{Str}(\hat{N}[J_1, \varepsilon \hat{\lambda}]) = \text{Str}(-J_1 \varepsilon [\hat{\lambda} \hat{\lambda}, \hat{w}]) = 0 , \quad (2.14)$$

which follow from the pure spinor conditions  $\{\lambda, \lambda\} = \{\hat{\lambda}, \hat{\lambda}\} = 0$ , we can derive

$$\varepsilon Q(S_{\text{gh}}) = \left\langle -J_3 \bar{\nabla}(\varepsilon \lambda) - \bar{J}_1 \nabla(\varepsilon \hat{\lambda}) \right\rangle . \quad (2.15)$$

Gathering all results obtained above together, we find that the BRST transformation of  $S$  is

$$\begin{aligned} \varepsilon Q(S) &= \frac{1}{4} \left\langle \nabla(\varepsilon \lambda \bar{J}_3 - \varepsilon \hat{\lambda} \bar{J}_1) - \bar{\nabla}(\varepsilon \lambda J_3 - \varepsilon \hat{\lambda} J_1) \right\rangle \\ &= \frac{1}{4} \left\langle \partial(\varepsilon \lambda \bar{J}_3 - \varepsilon \hat{\lambda} \bar{J}_1) - \bar{\partial}(\varepsilon \lambda J_3 - \varepsilon \hat{\lambda} J_1) \right\rangle . \end{aligned} \quad (2.16)$$

In the second equality, we have used the fact that  $\text{Str}([J_0, \varepsilon \lambda \bar{J}_3]) = 0$  and the similar relations.

We can conclude that  $S$  is BRST invariant as long as this surface term vanishes. For a closed string, the surface term always vanishes. For an open string, however, appropriate boundary conditions are required. In the next section we will examine these boundary conditions.

### 3 Boundary BRST invariance to D-brane configurations

In this section we will examine boundary conditions for the surface term to be eliminated, and show that they lead us to a classification of possible 1/2 supersymmetric D-brane configurations in the  $\text{AdS}_5 \times \text{S}^5$  background.

The surface term (2.16) turns to ¶

$$\begin{aligned}\varepsilon Q(S) &= \frac{1}{4\pi\alpha'} \int d^2\sigma \text{Str} \left[ \partial_\sigma (\varepsilon \lambda J_3 - \varepsilon \hat{\lambda} J_1)_\tau + \partial_\tau (\varepsilon \lambda J_3 - \varepsilon \hat{\lambda} J_1)_\sigma \right] \\ &= \frac{1}{4\pi\alpha'} \int d\tau \text{Str} \left[ (\varepsilon \lambda J_3 - \varepsilon \hat{\lambda} J_1)_\tau \right] \Big|_{\sigma=\sigma_*}\end{aligned}\quad (3.1)$$

where we have assumed that the surface term at  $\tau = \pm\infty$  vanishes as usual. The open string boundaries are at  $\sigma = \sigma_*$  with  $\sigma_* = 0, \pi$ . As seen in (A.21),  $J_{1\tau}$  and  $J_{3\tau}$  correspond to  $q_\alpha L_\tau^{1\alpha}$  and  $\hat{q}_{\hat{\alpha}} L_\tau^{2\hat{\alpha}}$ , respectively, where the corresponding Cartan one-form  $L^I$  is given in (A.18). It follows that  $J_{1\tau}$  and  $J_{3\tau}$  are polynomials in  $\theta^I$ . We should note that the surface terms do not cancel out each other. So we may examine each surface term separately without loss of generality. Our strategy is as follows. First we examine a few terms contained in  $J_1$  and  $J_3$ , and fix the boundary condition. Next we will show that the boundary condition we have fixed would eliminate all terms in (3.1).

First, we shall fix the boundary condition by examining the following three terms contained in  $J_{1\tau}$  and those in  $J_{3\tau}$

$$J_{1\tau} = q_\alpha L_\tau^{1\alpha} = q_\alpha \left( \partial_\tau \theta + \frac{1}{2} e_\tau^A \tilde{\gamma}^{0\dots 4} \gamma_A \hat{\theta} - \frac{i}{6} \tilde{\gamma}^{0\dots 4} \gamma_A \hat{\theta} (\theta \gamma^A \partial_\tau \theta + \hat{\theta} \gamma^A \partial_\tau \hat{\theta}) \right) + \dots, \quad (3.2)$$

$$J_{3\tau} = \hat{q}_{\hat{\alpha}} L_\tau^{2\hat{\alpha}} = \hat{q}_{\hat{\alpha}} \left( \partial_\tau \hat{\theta} - \frac{1}{2} e_\tau^A \tilde{\gamma}^{0\dots 4} \gamma_A \theta + \frac{i}{6} \tilde{\gamma}^{0\dots 4} \gamma_A \theta (\theta \gamma^A \partial_\tau \theta + \hat{\theta} \gamma^A \partial_\tau \hat{\theta}) \right) + \dots. \quad (3.3)$$

The first two terms in the most right-hand sides in the above equations come from  $D\theta$  defined in (A.19), while the last one is contained in  $\frac{m^2}{3!} D\theta$  where  $m^2$  is defined in (A.20). After fixing the boundary condition we shall show that the boundary condition eliminates all surface terms in section 4.

Substituting (3.2) and (3.3) into (3.1), we obtain the corresponding surface terms, which we shall denote as  $\delta_0 S$ ,

$$\begin{aligned}\delta_0 S &= \frac{1}{4\pi\alpha'} \int d\tau \varepsilon \left[ \lambda \gamma^{0\dots 4} \left( \partial_\tau \hat{\theta} - \frac{1}{2} e_\tau^A \tilde{\gamma}^{0\dots 4} \gamma_A \theta + \frac{i}{6} \tilde{\gamma}^{0\dots 4} \gamma_A \theta (\theta \gamma^A \partial_\tau \theta + \hat{\theta} \gamma^A \partial_\tau \hat{\theta}) \right) \right. \\ &\quad \left. + \hat{\lambda} \gamma^{0\dots 4} \left( \partial_\tau \theta + \frac{1}{2} e_\tau^A \tilde{\gamma}^{0\dots 4} \gamma_A \hat{\theta} - \frac{i}{6} \tilde{\gamma}^{0\dots 4} \gamma_A \hat{\theta} (\theta \gamma^A \partial_\tau \theta + \hat{\theta} \gamma^A \partial_\tau \hat{\theta}) \right) \right] \Big|_{\sigma=\sigma_*}\end{aligned}\quad (3.4)$$

where we have used  $\text{Str}(q_\alpha \hat{q}_{\hat{\beta}}) = (\gamma^{0\dots 4})_{\alpha\hat{\beta}}$  and  $\text{Str}(\hat{q}_{\hat{\alpha}} q_\beta) = -(\gamma^{0\dots 4})_{\hat{\alpha}\beta}$ . For our purpose, it is convenient to rewrite (3.4) in a 32-component notation. Defining

$$\boldsymbol{\lambda}^I \equiv \begin{pmatrix} \lambda^I \\ 0 \end{pmatrix}, \quad \boldsymbol{\theta}^I \equiv \begin{pmatrix} \theta^I \\ 0 \end{pmatrix}, \quad (3.5)$$

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¶  $\partial \bar{J} - \bar{\partial} J = \partial_\tau J_\sigma + \partial_\sigma J_\tau$ , as  $\partial = \partial_\tau + \partial_\sigma$  and  $\bar{\partial} = \partial_\tau - \partial_\sigma$ .

where  $(\lambda^1, \lambda^2) = (\lambda, \hat{\lambda})$  and  $(\theta^1, \theta^2) = (\theta, \hat{\theta})$ , and using the 32 component spinor notation given in Appendix A, we find that (3.4) may be simplified to

$$\delta_0 S = \frac{1}{4\pi\alpha'} \int d\tau \varepsilon \left[ \bar{\lambda} I \sigma_1 \partial_\tau \boldsymbol{\theta} + \frac{1}{2} e_\tau^A \bar{\lambda} \Gamma_A \sigma_3 \boldsymbol{\theta} - \frac{i}{6} \bar{\lambda} \Gamma_A \sigma_3 \boldsymbol{\theta} \bar{\theta} \Gamma^A \partial_\tau \boldsymbol{\theta} \right] \Big|_{\sigma=\sigma_*}. \quad (3.6)$$

To show this, the following relations are useful

$$\bar{\lambda}^I I(\sigma_1)_{IJ} \partial \boldsymbol{\theta}^J = \lambda^1 \gamma^{0\dots 4} \partial \theta^2 + \lambda^2 \gamma^{0\dots 4} \partial \theta^1, \quad (3.7)$$

$$\bar{\lambda}^I \Gamma_A(\sigma_3)_{IJ} \boldsymbol{\theta}^J = \lambda^1 \gamma_A \theta^1 - \lambda^2 \gamma_A \theta^2, \quad (3.8)$$

$$\bar{\theta} \Gamma^A \partial_\tau \boldsymbol{\theta} = \theta^1 \gamma^A \partial_\tau \theta^1 + \theta^2 \gamma^A \partial_\tau \theta^2. \quad (3.9)$$

For bosonic coordinates, we impose the boundary conditions as follows: Neumann boundary condition  $e_\sigma^{\bar{A}} = \partial_\sigma x^\mu e_\mu^{\bar{A}} = 0$  for  $\bar{A} = \bar{A}_0, \dots, \bar{A}_p$ , or Dirichlet boundary condition  $e_\tau^{\underline{A}} = \partial_\tau x^\mu e_\mu^{\underline{A}} = 0$  for  $\underline{A} = \underline{A}_{p+1}, \dots, \underline{A}_9$ . This boundary condition eliminates the surface term  $\delta x^\mu e_\mu^A e_\sigma^B \eta_{AB}$  at  $\sigma = \sigma_*$ . In order to delete  $\delta_0 S$ , we must impose boundary conditions on  $\boldsymbol{\theta}$  and  $\boldsymbol{\lambda}$ . The boundary condition we shall impose on  $\boldsymbol{\theta}$  is

$$\boldsymbol{\theta} = M \boldsymbol{\theta}, \quad M = s \Gamma^{\bar{A}_0 \dots \bar{A}_p} \otimes \rho \quad (3.10)$$

where  $\rho_{IJ} \in \{1, \sigma_1, i\sigma_2, \sigma_3\}$  is a two-by-two matrix acting on  $\boldsymbol{\theta}^I$  ( $I = 1, 2$ ). For the reality of  $\boldsymbol{\theta}$ , we choose  $s$  as  $s = \pm 1$ . The boundary condition leads to 1/2 supersymmetric D-branes. As  $\boldsymbol{\theta}^I$  are a pair of Majorana-Weyl spinors satisfying  $\boldsymbol{\theta} = \Gamma_{11} \boldsymbol{\theta}$ , we find  $p = \text{odd}$  for consistency,  $[M, \Gamma_{11}] = 0$ . For the boundary condition on  $\boldsymbol{\lambda}$ , we must impose  $\boldsymbol{\lambda} = M \boldsymbol{\lambda}$ . This is necessary for the BRST transformation to be non-trivial even at the boundary. In fact, the BRST transformation of  $J_1$  in (2.7), namely  $\varepsilon Q(\partial \theta^\alpha) = \varepsilon \partial \lambda^\alpha + \dots$ , is consistent if we impose the same boundary condition on  $\boldsymbol{\theta}$  and  $\boldsymbol{\lambda}$ .

We shall examine each term contained in (3.6) below so that we will fix  $p$  and  $\rho$ . Let us begin with examining the second term in the right hand side of (3.6). Because  $e_\tau^{\underline{A}} = 0$ ,

$$\bar{\lambda} \Gamma_{\bar{A}} \sigma_3 \boldsymbol{\theta} = 0 \quad (3.11)$$

must be satisfied. It follows from (3.11) that in order to delete the third term in the right hand side of (3.6),

$$\bar{\theta} \Gamma^{\bar{A}} \partial_\tau \boldsymbol{\theta} = 0 \quad (3.12)$$



must be satisfied. We examine (3.12) first. Noting that  $CM = \mp\alpha M^T C$  with  $\rho = \alpha\rho^T$  for  $p = \{ \frac{1}{3} \bmod 4$ , respectively, we derive

$$\bar{\theta}\Gamma^A\partial_\tau\theta = \bar{\theta}\Gamma^A M\partial_\tau\theta = \theta^T C M \Gamma^A \partial_\tau\theta = \mp\alpha \overline{M\theta}\Gamma^A\partial_\tau\theta = \mp\alpha \bar{\theta}\Gamma^A\partial_\tau\theta , \quad (3.13)$$

so that  $\alpha$  is fixed as  $\alpha = \pm 1$  for (3.12). It means that  $\rho = \pm\rho^T$  for  $p = \{ \frac{1}{3} \bmod 4$ .

Now, we return to (3.11). We derive, defining  $\beta$  by  $\sigma_3\rho = \beta\rho\sigma_3$ ,

$$\bar{\lambda}\Gamma_{\bar{A}}\sigma_3\theta = \bar{\lambda}\Gamma_{\bar{A}}\sigma_3 M\theta = \beta\bar{\lambda}\Gamma_{\bar{A}}M\sigma_3\theta = -\beta\bar{\lambda}M\Gamma_{\bar{A}}\sigma_3\theta = \beta\overline{M\lambda}\Gamma_{\bar{A}}\sigma_3\theta = \beta\bar{\lambda}\Gamma_{\bar{A}}\sigma_3\theta . \quad (3.14)$$

It implies that  $\beta$  is fixed as  $\beta = -1$  for (3.11). This means that  $\rho = \sigma_1$  or  $i\sigma_2$ . Combining this with the result obtained from (3.11), we can conclude that  $\rho = \sigma_1$  for  $p = 1 \bmod 4$ , and that  $\rho = i\sigma_2$  for  $p = 3 \bmod 4$ . For consistency we require that  $M^2 = 1$ . This implies that the time direction 0 is a Neumann direction since  $s^2 = 1$ . The results so far coincide with the boundary condition for 1/2 supersymmetric D-branes in flat spacetime.

Finally, we examine the first term in the right hand side of (3.6) which leads to the additional condition specific to the  $\text{AdS}_5 \times \text{S}^5$  background. One may show that

$$\bar{\lambda}I\sigma_1\partial_\tau\theta = \bar{\lambda}I\sigma_1 M\partial_\tau\theta = \pm\bar{\lambda}IM\sigma_1\partial_\tau\theta = \pm(-1)^n\bar{\lambda}MI\sigma_1\partial_\tau\theta = \mp(-1)^n\bar{\lambda}I\sigma_1\partial_\tau\theta . \quad (3.15)$$

In the second equality we have used  $\sigma_1\rho = \pm\rho\sigma_1$  for  $p = \{ \frac{1}{3} \bmod 4$ . The third equality follows from  $IM = (-1)^n MI$  where  $n$  is the number of Neumann directions contained in  $\text{AdS}_5$  spanned by  $\{0,1,2,3,4\}$ . As a result, for  $\bar{\lambda}I\sigma_1\partial_\tau\theta = 0$  we must impose  $n = \text{even}$  for  $p = 1 \bmod 4$  and  $n = \text{odd}$  for  $p = 3 \bmod 4$ . We summarize the result in the Table 1 where  $(n, n')$  means a D-brane of which world-volume is extended along  $\text{AdS}_n \times \text{S}^{n'}$ . This gives a classification of 1/2 supersymmetric D-brane configurations in the  $\text{AdS}_5 \times \text{S}^5$  background. This result is consistent with the ones obtained by using the  $\kappa$ -symmetry variation of the Green-Schwarz superstring [15, 18] and by examining  $Dp$ -brane field equations [14].

$p$	-1	1	3	5	7	9
D-brane	—	(2, 0)	(1, 3), (3, 1)	(2, 4), (4, 2)	(3, 5), (5, 3)	—

Table 1: 1/2 supersymmetric D-brane configurations in  $\text{AdS}_5 \times \text{S}^5$

Summarizing the results, we find that the surface term (3.6) vanishes if we impose the boundary conditions

$$\theta = M\theta , \quad \lambda = M\lambda , \quad (3.16)$$

with

$$M = \begin{cases} s\Gamma^{\bar{A}_0 \dots \bar{A}_p} \otimes \sigma_1 \\ s\Gamma^{\bar{A}_0 \dots \bar{A}_p} \otimes i\sigma_2 \end{cases}, \quad n = \begin{cases} \text{even} \\ \text{odd} \end{cases} \quad \text{for } p = \begin{cases} 1 \\ 3 \end{cases} \pmod{4}, \quad s = \pm 1 \quad (3.17)$$

respectively.

## 4 Proof of validity in eliminating all surface terms

In the previous section, we have derived the boundary conditions (3.16) with (3.17) which eliminate a certain terms (3.6) contained in the surface term (3.1). In this section, we shall show that the boundary conditions fixed in the previous section may eliminate all terms contained in the surface term (3.1).

For this purpose, it is convenient to rewrite the surface term (3.1) in the 32-component notation as

$$\varepsilon Q(S) = -\frac{\varepsilon}{4\pi\alpha'} \int d\tau [\bar{\lambda} I \sigma_1 \mathbf{L}_\tau] \big|_{\sigma=\sigma_*} \quad (4.1)$$

where the corresponding Cartan one-form  $\mathbf{L}$  is given in (A.6). We will show that the validity of the boundary conditions for  $p = 1 \pmod{4}$  and for  $p = 3 \pmod{4}$ , in turn.

### 4.1 $p = 1 \pmod{4}$

We shall show that the surface term (4.1) is eliminated by the boundary conditions for  $p = 1 \pmod{4}$ :  $\boldsymbol{\theta} = P_+ \boldsymbol{\theta}$  and  $\boldsymbol{\lambda} = P_+ \boldsymbol{\lambda}$  where  $P_+ = \frac{1}{2}(1 + M)$  with  $M = s\Gamma^{\bar{A}_0 \dots \bar{A}_p} \otimes \sigma_1$  and  $n = \text{even}$ .

First we examine  $D_\tau \boldsymbol{\theta}$  defined in (A.7). It follows from  $\boldsymbol{\theta} = P_+ \boldsymbol{\theta}$  that

$$D_\tau \boldsymbol{\theta} = P_+ D_\tau \boldsymbol{\theta}. \quad (4.2)$$

In order to derive this relation, we have used

$$\partial_\tau \boldsymbol{\theta} = P_+ \partial_\tau \boldsymbol{\theta}, \quad (4.3)$$

$$\frac{1}{2} e_\tau^{\bar{A}} I \Gamma_{\bar{A}} \epsilon \boldsymbol{\theta} = \frac{1}{2} e_\tau^{\bar{A}} I \Gamma_{\bar{A}} \epsilon P_+ \boldsymbol{\theta} = \frac{1}{2} e_\tau^{\bar{A}} I \Gamma_{\bar{A}} P_- \epsilon \boldsymbol{\theta} = \frac{1}{2} e_\tau^{\bar{A}} I P_+ \Gamma_{\bar{A}} \epsilon \boldsymbol{\theta} = P_+ \frac{1}{2} e_\tau^{\bar{A}} I \Gamma_{\bar{A}} \epsilon \boldsymbol{\theta}, \quad (4.4)$$

$$\frac{1}{4} w_\tau^{AB} \Gamma_{AB} \boldsymbol{\theta} = \frac{1}{4} w_\tau^{\bar{A}\bar{B}} \Gamma_{\bar{A}\bar{B}} P_+ \boldsymbol{\theta} + \frac{1}{4} w_\tau^{\underline{A}\underline{B}} \Gamma_{\underline{A}\underline{B}} P_+ \boldsymbol{\theta} = P_+ \left( \frac{1}{4} w_\tau^{\bar{A}\bar{B}} \Gamma_{\bar{A}\bar{B}} \boldsymbol{\theta} + \frac{1}{4} w_\tau^{\underline{A}\underline{B}} \Gamma_{\underline{A}\underline{B}} \boldsymbol{\theta} \right). \quad (4.5)$$

Here we have assumed that  $w^{\bar{A}\bar{B}} = 0$ . This is because a D-brane breaks rotational invariance in the plane spanned by one of Neumann directions and one of Dirichlet directions.

Next we will examine  $\mathcal{M}^2 P_+$  where  $\mathcal{M}^2$  is defined in (A.8). Noting that  $CP_\pm = P_\mp^T C$  one derives

$$-iI\Gamma_A \epsilon \bar{\theta} \Gamma^A P_+ = -iI\Gamma_{\bar{A}} \epsilon \bar{\theta} P_- \Gamma^{\bar{A}} P_+ = -iP_+ IP_+ \Gamma_{\bar{A}} P_- \epsilon P_+ \bar{\theta} P_- \Gamma^{\bar{A}} P_+ , \quad (4.6)$$

$$\begin{aligned} \frac{i}{2} \Gamma_{AB} \bar{\theta} \hat{\Gamma}^{AB} \epsilon P_+ &= \frac{i}{2} \Gamma_{AB} \bar{\theta} \hat{\Gamma}^{AB} P_- \epsilon P_+ \\ &= \frac{i}{2} \Gamma_{\bar{A}\bar{B}} \bar{\theta} P_- \hat{\Gamma}^{\bar{A}\bar{B}} P_- \epsilon P_+ + \frac{i}{2} \Gamma_{\underline{AB}} \bar{\theta} P_- \hat{\Gamma}^{\underline{AB}} P_- \epsilon P_+ \\ &= P_+ \frac{i}{2} \Gamma_{\bar{A}\bar{B}} \bar{\theta} P_- \hat{\Gamma}^{\bar{A}\bar{B}} P_- \epsilon P_+ + P_+ \frac{i}{2} \Gamma_{\underline{AB}} \bar{\theta} P_- \hat{\Gamma}^{\underline{AB}} P_- \epsilon P_+ . \end{aligned} \quad (4.7)$$

It follows that

$$\mathcal{M}^2 P_+ = P_+ \mathcal{M}^2 P_+ . \quad (4.8)$$

Gathering the results (4.2) and (4.8) together we obtain  $\mathbf{L}_\tau = P_+ \mathbf{L}_\tau$ . Using this we may derive

$$\bar{\lambda} I \sigma_1 \mathbf{L}_\tau = \bar{\lambda} I \sigma_1 P_+ \mathbf{L}_\tau = \bar{\lambda} P_+ IP_+ \sigma_1 P_+ \mathbf{L}_\tau = \overline{P_- \bar{\lambda}} P_+ IP_+ \sigma_1 P_+ \mathbf{L}_\tau = 0 . \quad (4.9)$$

This shows that the boundary conditions for  $p = 1 \bmod 4$  eliminate the surface term (4.1).

## 4.2 $p = 3 \bmod 4$

We show that the surface term (4.1) is eliminated by the boundary conditions for  $p = 3 \bmod 4$ :  $\theta = P_+ \theta$  and  $\lambda = P_+ \lambda$  where  $P_+ = \frac{1}{2}(1 + M)$  with  $M = \Gamma^{\bar{A}_0 \dots \bar{A}_p} \otimes i\sigma_2$  and  $n = \text{odd}$ .

First we examine  $D_\tau \theta$ . The calculation similar to the one in the case with  $p = 1 \bmod 4$ , except for

$$\frac{1}{2} e_\tau^{\bar{A}} I \Gamma_{\bar{A}} \epsilon \theta = \frac{1}{2} e_\tau^{\bar{A}} I \Gamma_{\bar{A}} \epsilon P_+ \theta = \frac{1}{2} e_\tau^{\bar{A}} I \Gamma_{\bar{A}} P_+ \epsilon \theta = \frac{1}{2} e_\tau^{\bar{A}} I P_- \Gamma_{\bar{A}} \epsilon \theta = P_+ \frac{1}{2} e_\tau^{\bar{A}} I \Gamma_{\bar{A}} \epsilon \theta , \quad (4.10)$$

leads us to

$$D_\tau \theta = P_+ D_\tau \theta . \quad (4.11)$$

Next we will examine  $\mathcal{M}^2 P_+$ . Noting that  $CP_\pm = P_\mp^T C$  one derives

$$-iI\Gamma_A \epsilon \bar{\theta} \Gamma^A P_+ = -iP_+ IP_- \Gamma_{\bar{A}} P_+ \epsilon P_+ \bar{\theta} P_- \Gamma^{\bar{A}} P_+ , \quad (4.12)$$

$$\frac{i}{2} \Gamma_{AB} \bar{\theta} \hat{\Gamma}^{AB} \epsilon P_+ = P_+ \frac{i}{2} \Gamma_{\bar{A}\bar{B}} \bar{\theta} P_- \hat{\Gamma}^{\bar{A}\bar{B}} P_+ \epsilon P_+ + P_+ \frac{i}{2} \Gamma_{\underline{AB}} \bar{\theta} P_- \hat{\Gamma}^{\underline{AB}} P_+ \epsilon P_+ , \quad (4.13)$$

so that

$$\mathcal{M}^2 P_+ = P_+ \mathcal{M}^2 P_+ . \quad (4.14)$$

Gathering the results (4.11) and (4.14) together we obtain  $\mathbf{L}_\tau = P_+ \mathbf{L}_\tau$ . Using this we may derive

$$\bar{\lambda} I \sigma_1 \mathbf{L}_\tau = \bar{\lambda} I \sigma_1 P_+ \mathbf{L}_\tau = \bar{\lambda} P_+ I P_- \sigma_1 P_+ \mathbf{L}_\tau = \overline{P_- \lambda} P_+ I P_- \sigma_1 P_+ \mathbf{L}_\tau = 0 . \quad (4.15)$$

It implies that the boundary condition for  $p = 3 \bmod 4$  eliminates the surface term (4.1).

Summarizing we have shown that the boundary condition (3.16) with (3.17) eliminates the surface term (3.1) of the BRST transformation  $\varepsilon Q(S)$ .

## 5 Summary and discussions

We examined the BRST invariance of the open pure spinor superstring action in the  $\text{AdS}_5 \times \text{S}^5$  background. In order for the BRST symmetry to be preserved even in the presence of the boundary, the surface term of the BRST transformation must be eliminated by appropriate boundary conditions. We determined such boundary conditions and found that the boundary conditions lead to a classification of possible configurations of 1/2 supersymmetric D-branes in the  $\text{AdS}_5 \times \text{S}^5$  background. Our result is summarized in the Table 1. This is consistent with the results obtained by the other approaches [14, 15, 18].

We have used an exponential parametrization of the coset representative  $g$  throughout this paper. In [21] a GS superstring action in the  $\text{AdS}_5 \times \text{S}^5$  background was derived based on an alternate version of the coset superspace construction in terms of  $\text{GL}(4|4)$ . The pure spinor superstring action in this coset superspace construction was given in [22]. This action is expected to make it more transparent to examine the surface term for the BRST transformation of the action and to derive possible D-brane configurations in the  $\text{AdS}_5 \times \text{S}^5$  background.

The method used in this paper can be applied easily to a superstring in the other background, for example the superstring in the type IIB pp-wave background [23][24]. The result will be consistent with the one obtained by the boundary  $\kappa$ -invariance of the open GS superstring [15, 20] and by examining equations of motion for a D-brane [14].

It is also known that in the presence of a constant flux, the boundary condition to ensure the  $\kappa$ -invariance of the GS superstring action leads to possible (non-commutative) D-branes [25]. Furthermore the boundary condition to ensure the  $\kappa$ -invariance of the supermembrane action leads to the self-duality condition for the three-form flux on the M5-brane world-volume [26]. The same result is expected to be obtained by using the pure spinor supermembrane action [27]. We hope to report this issue in another place [28].

Finally let us comment on a characterization of the Wess-Zumino (WZ) action. The WZ action is necessary for the  $\kappa$ -symmetry of the action, and then halves fermionic degrees of freedom on the world-volume so as to match bosonic and fermionic degrees of freedom. It is shown that the WZ term of a (D) $p$ -brane in flat spacetime is characterized as a non-trivial element of the Chevalley-Eilenberg (CE) cohomology in [29] for  $p$ -branes, and in [30, 31] for  $Dp$ -branes. In [32],  $Dp$ -brane actions in the extended pure spinor formalism [2] are characterized as a non-trivial element of the BRST cohomology of the extended BRST symmetry. It is interesting for us to extend this analysis to the  $Dp$ -brane action in the  $\text{AdS}_5 \times \text{S}^5$  background.<sup>||</sup>

## Acknowledgments

The authors would like to thank Takanori Fujiwara, Yoshifumi Hyakutake and Kentaroh Yoshida for useful comments.

## Appendix

### A Notation and convention

The super-isometry algebra of the  $\text{AdS}_5 \times \text{S}^5$  background is  $\mathfrak{psu}(2, 2|4)$  of which (anti-)commutation relations are \*\*

$$\begin{aligned} [P_a, P_b] &= M_{ab} , & [P_{a'}, P_{b'}] &= -M_{a'b'} , \\ [M_{AB}, P_C] &= \eta_{BC} P_A - \eta_{AC} P_B , & [M_{AB}, M_{CD}] &= \eta_{BC} M_{AD} + 3\text{-terms} , \end{aligned} \quad (\text{A.1})$$

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<sup>||</sup> On a CE cohomology classification of  $Dp$ -brane actions in the  $\text{AdS}_5 \times \text{S}^5$  background see e.g. [33, 34].

\*\*We follow the notation given in [35] except for  $\Gamma_{11}$ .

and

$$\begin{aligned} [Q_I, M_{AB}] &= -\frac{1}{2}Q_I\Gamma_{AB}, \quad [Q_I, P_A] = \frac{1}{2}\epsilon_{IJ}Q_J\Gamma_A, \\ \{Q_I, Q_J\} &= -2iC\Gamma^AP_A\delta_{IJ}h_+ + \epsilon_{IJ}\left(iC\Gamma^{ab}IM_{ab} - iC\Gamma^{a'b'}IM_{a'b'}\right)h_+, \end{aligned} \quad (\text{A.2})$$

where  $\Gamma^A$  ( $A = 0, 1, \dots, 9$ ) are  $32 \times 32$  gamma matrices, and  $Q_I = Q_I h_+$  ( $I = 1, 2$ ) are a pair of Majorana-Weyl spinors. We introduced  $\epsilon_{IJ} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ . We have defined  $I = \Gamma^{01234}$  and  $h_{\pm} = \frac{1}{2}(1 \pm \Gamma_{11})$  with  $\Gamma_{11} \equiv \Gamma^{012\dots 9}$ . The charge conjugation matrix  $C$  satisfies  $C\Gamma_A = -\Gamma_A^T C$ . The  $\text{AdS}_5$  isometry is generated by  $P_a$  and  $M_{ab}$  ( $a, b = 0, 1, \dots, 4$ ), while the  $S^5$  isometry is by  $P_{a'}$  and  $M_{a'b'}$  ( $a', b' = 5, 6, \dots, 9$ ).

The left-invariant Cartan one-form  $\mathbf{L}$  is defined by

$$\mathbf{L} = g^{-1}dg = \mathbf{L}^A P_A + \frac{1}{2}\mathbf{L}^{AB}M_{AB} + Q_I \mathbf{L}^I \quad (\text{A.3})$$

where  $g \in \text{PSU}(2,2|4)/(\text{SO}(1,4) \times \text{SO}(5))$ . Parametrizing  $g$  by  $g = g_x(x)e^{Q_I \theta^I}$  and introducing the vielbein  $e^A(x)$  and spin-connection  $w^{AB}(x)$  by  $g_x^{-1}dg_x = e^A P_A + \frac{1}{2}w^{AB}M_{AB}$ , we obtain

$$\mathbf{L}^A = e^A - 2i\bar{\theta}\Gamma^A\left(\frac{1}{2} + \frac{\mathcal{M}^2}{4!} + \frac{\mathcal{M}^4}{6!} + \dots\right)D\theta, \quad (\text{A.4})$$

$$\mathbf{L}^{AB} = w^{AB} + 2i\bar{\theta}\hat{\Gamma}^{AB}\epsilon\left(\frac{1}{2} + \frac{\mathcal{M}^2}{4!} + \frac{\mathcal{M}^4}{6!} + \dots\right)D\theta, \quad (\text{A.5})$$

$$\mathbf{L}^I = \left(1 + \frac{\mathcal{M}^2}{3!} + \frac{\mathcal{M}^4}{5!} + \dots\right)D\theta, \quad (\text{A.6})$$

where we have defined  $\bar{\theta} \equiv \theta^T C$ ,  $\hat{\Gamma}^{AB} \equiv (\Gamma^{ab}I, -\Gamma^{a'b'}I)$  and

$$D\theta = d\theta + \frac{1}{2}e^A\Gamma_A\epsilon\theta + \frac{1}{4}w^{AB}\Gamma_{AB}\theta, \quad (\text{A.7})$$

$$\mathcal{M}^2 = -i\Gamma_A\epsilon\theta \cdot \bar{\theta}\Gamma^A + \frac{i}{2}\Gamma_{AB}\theta \cdot \bar{\theta}\hat{\Gamma}^{AB}\epsilon. \quad (\text{A.8})$$

## A.1 16-component spinor notation

Let us rewrite (A.2) in terms of  $16 \times 16$  gamma-matrices  $\gamma^A$ . We decompose  $\Gamma^A$  as

$$\Gamma^0 = \mathbf{1}_{16} \otimes i\sigma_2, \quad \Gamma^i = \gamma^i \otimes \sigma_1, \quad \Gamma^9 = \gamma \otimes \sigma_1 \quad (\text{A.9})$$

where  $\gamma \equiv \gamma^{1\dots 8}$  and  $i = 1, 2, \dots, 8$ . It implies that <sup>††</sup>

$$\Gamma^A = \begin{pmatrix} 0 & \tilde{\gamma}^A \\ \gamma^A & 0 \end{pmatrix}, \quad \tilde{\gamma}^A \equiv (1, \gamma^i, \gamma), \quad \gamma^A \equiv (-1, \gamma^i, \gamma). \quad (\text{A.10})$$

The anti-commutation relation  $\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}\mathbf{1}_{32}$  turns out to

$$\tilde{\gamma}^A \gamma^B + \tilde{\gamma}^B \gamma^A = 2\eta^{AB}\mathbf{1}_{16}, \quad \gamma^A \tilde{\gamma}^B + \gamma^B \tilde{\gamma}^A = 2\eta^{AB}\mathbf{1}_{16}. \quad (\text{A.11})$$

It follows that

$$\begin{aligned} \Gamma_{AB} &= \begin{pmatrix} \tilde{\gamma}_{AB} & 0 \\ 0 & \gamma_{AB} \end{pmatrix}, \quad I = \begin{pmatrix} 0 & \tilde{\gamma}^{0\dots 4} \\ \gamma^{0\dots 4} & 0 \end{pmatrix}, \quad I\Gamma_A = \begin{pmatrix} \tilde{\gamma}^{0\dots 4}\gamma_A & 0 \\ 0 & \gamma^{0\dots 4}\tilde{\gamma}_A \end{pmatrix}, \\ \Gamma_{11} &= \begin{pmatrix} \tilde{\gamma}^0\gamma^1\dots\gamma^9 & 0 \\ 0 & \gamma^0\tilde{\gamma}^1\dots\gamma^9 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad C = \Gamma_0^\dagger = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \\ C\Gamma^A &= \begin{pmatrix} \gamma^A & 0 \\ 0 & -\tilde{\gamma}^A \end{pmatrix}, \quad C\Gamma^{AB}I = \begin{pmatrix} \gamma^{AB}\gamma^{0\dots 4} & 0 \\ 0 & -\tilde{\gamma}^{AB}\tilde{\gamma}^{0\dots 4} \end{pmatrix}, \end{aligned} \quad (\text{A.12})$$

where we have defined the following objects

$$\begin{aligned} \tilde{\gamma}_{AB} &\equiv \frac{1}{2}(\tilde{\gamma}_A\gamma_B - \tilde{\gamma}_B\gamma_A), \quad \gamma_{AB} \equiv \frac{1}{2}(\gamma_A\tilde{\gamma}_B - \gamma_B\tilde{\gamma}_A), \\ \tilde{\gamma}^{0\dots 4} &= \tilde{\gamma}^0\gamma^1\tilde{\gamma}^2\gamma^3\tilde{\gamma}^4, \quad \gamma^{0\dots 4} = \gamma^0\tilde{\gamma}^1\gamma^2\tilde{\gamma}^3\gamma^4. \end{aligned} \quad (\text{A.13})$$

As  $I^2 = -1$ , we have  $\gamma^{0\dots 4}\tilde{\gamma}^{0\dots 4} = \tilde{\gamma}^{0\dots 4}\gamma^{0\dots 4} = -1$ .  $Q_I h_+ = Q_I$  implies that  $Q_I = (q_I, 0)$  with  $q_I$  ( $I = 1, 2$ ) being a pair of 16-component spinors. Similarly,  $h_+ \boldsymbol{\theta} = \boldsymbol{\theta}$  implies that  $\boldsymbol{\theta}^I = \begin{pmatrix} \theta^I \\ 0 \end{pmatrix}$  with  $\theta^I$  ( $I = 1, 2$ ) being a pair of 16-component spinors. By using  $q_I$ , (A.2) takes of the form

$$\begin{aligned} [q_I, M_{AB}] &= -\frac{1}{2}q_I \tilde{\gamma}_{AB}, \quad [q_I, P_A] = \frac{1}{2}\epsilon_{IJ}q_J \tilde{\gamma}^{0\dots 4}\gamma_A, \\ \{q_I, q_J\} &= -2i\gamma^A P_A \delta_{IJ} + \epsilon_{IJ}(i\gamma^{ab}\gamma^{0\dots 4}M_{ab} - i\gamma^{a'b'}\gamma^{0\dots 4}M_{a'b'}). \end{aligned} \quad (\text{A.14})$$

The fact that  $C\Gamma^{A_1\dots A_n}$  is symmetric iff  $n = 1, 2 \bmod 4$  implies that  $\gamma_A, \tilde{\gamma}_A, \gamma_{A_1\dots A_5}$  and  $\tilde{\gamma}_{A_1\dots A_5}$  are symmetric and that  $\gamma_{ABC}$  and  $\tilde{\gamma}_{ABC}$  are antisymmetric.

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<sup>††</sup> In the literature,  $\gamma^A$  and  $\tilde{\gamma}^A$  are distinguished each other by putting spinor indices. In this paper, however, we use a matrix notation without spinor indices to make our presentation simpler.

In this notation, the left-invariant Cartan one-forms are given as

$$L = L^A P_A + \frac{1}{2} L^{AB} M_{AB} + q_I L^I , \quad (\text{A.15})$$

$$L^A = e^A - 2i\theta\gamma^A \left( \frac{1}{2} + \frac{m^2}{4!} + \frac{m^4}{6!} + \dots \right) D\theta , \quad (\text{A.16})$$

$$L^{AB} = w^{AB} + 2i\theta\hat{\gamma}^{AB}\epsilon \left( \frac{1}{2} + \frac{m^2}{4!} + \frac{m^4}{6!} + \dots \right) D\theta , \quad (\text{A.17})$$

$$L^I = \left( 1 + \frac{m^2}{3!} + \frac{m^4}{5!} + \dots \right) D\theta , \quad (\text{A.18})$$

where  $\hat{\gamma}^{AB} = (\gamma^{ab}\gamma^{0\dots 4}, -\gamma^{a'b'}\gamma^{0\dots 4})$  and

$$D\theta = d\theta + \frac{1}{2} e^A \tilde{\gamma}^{0\dots 4} \gamma_A \epsilon \theta + \frac{1}{4} w^{AB} \tilde{\gamma}_{AB} \theta , \quad (\text{A.19})$$

$$m^2 = -i\tilde{\gamma}^{0\dots 4} \gamma_A \epsilon \theta \cdot \theta \gamma^A + \frac{i}{2} \tilde{\gamma}_{AB} \theta \cdot \theta \epsilon \hat{\gamma}^{AB} . \quad (\text{A.20})$$

The currents used in (2.5) are related to the above objects by

$$J_{0\xi} = \frac{1}{2} L_\xi^{AB} M_{AB} , \quad J_{1\xi} = q_\alpha L_\xi^{1\alpha} , \quad J_{2\xi} = L_\xi^A P_A , \quad J_{3\xi} = \hat{q}_{\hat{\alpha}} L_\xi^{2\hat{\alpha}} , \quad (\text{A.21})$$

where we have replaced  $(q^1, q^2)$  with  $(q_\alpha, \hat{q}_{\hat{\alpha}})$  and correspondingly  $(\theta^1, \theta^2)$  with  $(\theta^\alpha, \hat{\theta}^{\hat{\alpha}})$ .

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